DIOPHANTINE ANALYSIS.

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SOLUTIONS OF PROBLEMS.

18. Proposed by Professor G. B. M. ZERR, A. M., Principal of High School, Staunton, Virginia.

Decompose into the sum of two squares the number $17^3.73^5$.

I. Solution by the PROPOSER.

$$(a^2+b^2)(a^2+b^2)=(a^2\pm b^2)^2+(ab\mp ab)^2=A^2+B^2.$$

$$(a^2+b^2)^3=(A^2+B^2)(a^2+b^2)=(Aa\pm Bb)^2+(Ab\mp Ba)^2.$$

$$\therefore\quad (a^2+b^2)^3=\{a(a^2-3b^2)\}^2+\{b(3a^2-b^2)\}^2,$$
$$=\{a(a^2+b^2)\}^2+\{b(a^2+b^2)\}^2.$$

Similarly $(c^2+d^2)^5=\{c(c^4-10c^2d^2+5d^4)\}^2+\{d(5c^4-10c^2d^2+d^4)\}^2,$

$$=\{c(c^2+d^2)^2\}^2+\{d(c^2+d^2)^2\}^2,$$
$$=\{c(c^2+d^2)(3d^2-c^2)\}^2+d(c^2+d^2)(3c^2-d^2)\}^2.$$

Let $a=4$, $b=1$, $c=8$, $d=3$. $\therefore\quad 17^3=52^2+47^2=68^2+17^2$.

$$73^5=10072^2+44403^2=42632^2+15987^2=21608^2+40077^2,$$

$$\therefore\ 17^3.73^5=3092572^2+788035^2=2357900^2+2150653^2$$
$$=362372^2+3170755^2=1811860^2+2627197^2$$
$$=3190628^2+69955^2=2848180^2+1439747^2$$
$$=3099580^2+560003^2=1068428^2+3007235^2$$
$$=2835028^2+1465475^2=1172380^2+2968253^2$$
$$=2782340^2+1565197^2=1835572^2+2612685^2.$$

$\therefore$ the sum of two squares in twelve ways.

II. Solution by H. W. DRAUGHON, Olio, Mississippi. and O. W. ANTHONY. Missouri Military Academy, Mexico, Missouri.

Since $17^3\times73^5=17^2\times73^4\times17\times73=17^2\times73^4\times1241$.

And $1241=35^2+4^2=29^2+20^2$. Therefore,

$$17^3\times73^5=(17\times73^2\times35)^2+(17\times73^2\times4)^2=(17\times73^2\times29)^2+(17\times73^2\times20)^2.$$

III. Solution by COOPER D. SCHMITT, A. M., Professor of Mathematics, University of Tennessee, Knoxville, Tennessee.

Solution by determenants,

$$17=\begin{vmatrix}4 & -1\\ 1 & 4\end{vmatrix},\qquad 73=\begin{vmatrix}8 & -3\\ 3 & 8\end{vmatrix}\qquad 17^2=\begin{vmatrix}17 & 0\\ 0 & 17\end{vmatrix}\qquad 73^4=\begin{vmatrix}5329 & 0\\ 0 & 5329\end{vmatrix}$$

$$17^3=\begin{vmatrix}17 & 0\\ 0 & 17\end{vmatrix}\times\begin{vmatrix}4 & -1\\ 1 & 4\end{vmatrix}=\begin{vmatrix}68 & 17\\ -17 & 68\end{vmatrix}$$

$$17^3\times73=\begin{vmatrix}68 & 17\\ -17 & 68\end{vmatrix}\times\begin{vmatrix}8 & -3\\ 3 & 8\end{vmatrix}=\begin{vmatrix}344-51 & 204+136\\ -136-204 & -51+344\end{vmatrix}=\begin{vmatrix}493 & 340\\ -340 & 493\end{vmatrix}.$$

$$\text{Then } 17^3\times73^5=\begin{vmatrix}493 & 340\\ -340 & 493\end{vmatrix}\times\begin{vmatrix}5329 & 0\\ 0 & 5329\end{vmatrix}=(493^2+340^2)(5329)^2$$

$=(73)^2(493)^2+(73)^2(340)^2$, which is one set of answers.

Also solved by *R. J. Adcock*, and *M. A. Gruber*.

19. Proposed by ARTEMAS MARTIN, LL. D., U. S. Coast and Geodetic Survey Office, Washington, D. C

Find three positive integer numbers whose sum is a cube, and, also, the sum of any two diminished by the third a cube.

I. Solution by M. A. GRUBER, A. M., War Department, Washington, D. C.

Let x, y, and z=the three positive integers.

$$\text{Then } \begin{aligned} x+y+z&=a^3\\ x+y-z&=b^3\\ x+z-y&=c^3\\ y+z-x&=d^3\end{aligned}$$

$$\text{Whence } x+y+z=b^3+c^3+d^3=a^3;$$

$$x=\frac{b^3+c^3}{2}\ ;\ y=\frac{b^3+d^3}{2}\ ;\ z=\frac{c^3+d^3}{2}\ .$$

This is a problem in which the sum of three cubes=a cube. Take $3^3+4^3+5^3=6^3$. But as the numbers are to be integers, we multiply by 2, and obtain $6^3+8^3+10^3=12^3$. $\therefore$ $x=\dfrac{6^3+8^3}{2}=364$; $y=\frac{1}{2}(6^3+10^3)=608$; and $z=\frac{1}{2}(8^3+10^3)=756$. The number of answers is infinite.

II. Solution by R. J. ADCOCK, Larchland, Warren County, Illinois.

Let x, y, z, be the three numbers; then $x+y+z=u^3$, $x+y-z=v^3$, $x+z-y=r^3$, $z+y-x=s^3$, by conditions.

Wherefore $x+y+l=u^3=v^3+r^3+s^3$, $x=\dfrac{v^3+r^3}{2}$, $y=\frac{1}{2}(v^3+s^3)$, $z=\frac{1}{2}(r^3+s^3)$. The most general equation yet obtained by me for the sum of three cubes=a cube, is found from,

$[(ax^3+dy^3)x]^3+[(bx^3+hy^3)y]^3+[(cx^3-hy^3)y]^3=[(ax^3+gy^3)x]^3$, by expanding, equating coefficients of similar terms with respect to x and y, eliminating d, g, and h, giving the identical equation,

$$[9a^3bx^3y+(b^2-bc+c^2)^2y^4]^3+[9a^3cx^3y-(b^2-bc+c^2)^2y^4]$$
$$+[9a^4x^4-3(b^2-bc+c^2)caxy^3]^3=[9a^4x^3+3ab(b^2-bc+c^2)xy^3]^3.$$

By numbers for the letters in the above, some of the resulting